King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

Information and Computer Science Department

ICS 253: Discrete Structures I

Summer Semester 2012-2013

Major Exam #1, Saturday June 22, 2013.

Name:

ID#:

**Instructions**:

1. This exam consists of **seven** pages, including this page, containing **four** questions.
2. You have to answer all **four** questions.
3. The exam is closed book and closed notes. No calculators or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
4. The questions are **NOT equally weighed**. Some questions count for more points than others.
5. The maximum number of points for this exam is **100**.
6. You have exactly **90** minutes to finish the exam.
7. Make sure your answers are **readable**.
8. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order for me not to miss grading it.

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| --- | --- | --- |
| Question Number | Maximum # of Points | Earned Points |
| 1 | **40** |  |
| 2 | **20** |  |
| 3 | **25** |  |
| 4 | **15** |  |
| **Total** | **100** |  |

|  |  |  |  |
| --- | --- | --- | --- |
|  | Addition |  | Modus Tollens |
|  | Simplification |  | Hypothetical syllogism |
|  | Conjunction |  | Disjunctive syllogism |
|  | Modus Ponens |  | Resolution |

1. **(40 points) Propositional Logic and Propositional Equivalences**
	1. (9 points) Construct a truth table for the compound proposition $\left(\left(p⟶q\right)⟶r\right)⟶s$.
	2. (1 points) Determine whether the compound proposition in “a.” is a tautology, contradiction or contingency.
	3. (6 points) State the converse, contrapositive and inverse of the conditional statement: You get an A in the course unless you do really bad in the final.
	4. (10 points) Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Clearly explain your reasoning.
	5. (14 points) Show that $\left[\left(p∨q\right)∧\left(p⟶r\right)∧\left(q⟶r\right)\right]⟶r$ is a tautology **without** using a truth table.
2. **(20 points) Predicates and Quantifiers**

Let I(x) be the statement “x has an Internet connection” and C(x, y) be the statement “x and y have chatted over the Internet,” where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements, where no negation is outside a quantifier or an expression involving logical connectives. You are **not** allowed to us “∃!”.

* 1. (2 points) Fahad and Salem have never chatted over the Internet.
	2. (2 points) No one in the class has chatted with Salem.
	3. (4 points) Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.
	4. (4 points) Someone in your class has an Internet connection but has not chatted with anyone else in your class.
	5. (4 points) Everyone except exactly one student in your class has an Internet connection.
	6. (4 points) There are two students in your class who have not chatted with each other over the Internet.
1. **(25 points) Rules of Inference and Methods of Proof**
	1. (5 points) Represent the following argument using predicates and quantifiers and then determine it is correct or incorrect. Clearly explain why: “All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.”
	2. (5 points) Prove that the sum of a rational number and an irrational number is an irrational number.
	3. (15 points) Show that these statements about the integer *x* are equivalent: (*i*) 3*x* + 2 is even, (*ii*) *x* + 5 is odd, (*iii*) *x*2 is even.
2. **(15 points) Sets, Set Operations and Functions**
	1. (3 points) Let *A* = {$ *,* #}, B = {*x*, *y*}, and *C* = {0, 1}.

Find *C* × *B* × *A*.

* 1. (6 points) Find the truth set of each of these predicates where the domain is the set of integers.
		1. (2 points) *P*(x): "$x^{2}<20$"
		2. (2 point) *Q*(x): "$x^{2}>-x$"
		3. (2 points) *R*(x): "$2x-2=0$"
	2. (3 points) What is the cardinality of each of the following sets
		1. {a, {a}, {a, {a}}}
		2. *P*({ Φ, a, {a}, {{a}}})
	3. (3 points) Draw the Venn diagram for $\left(A∩B\right)∪\left(A∩C\right)$